MODELS FOR MAKING DECISIONS ON PURCHASE VOLUMES DEPENDING ON THE ASSESSMENT OF UPCOMING RETAIL SALES

Zalozhnev A. Yu.[1],
Chistov D. V.[2]

Abstract: This article considers an economic system that consists of three types of business entities: a manufacturer, a distributor (wholesaler), and a retailer. Mathematical models that allow the distributor (wholesaler) to make informed decisions about the volume of goods purchased from the manufacturer are considered, using the assessment of upcoming retail sales, assuming that the manufacturer's selling price depends on the volume of purchases. Models are built from the standpoint of resolving conflicts between participants in the decision-making process: purchasing managers who strive to have enough inventory and financiers who want to reduce this volume, for example, to increase the liquidity of the organization's assets or reduce storage costs. The constructed models allow the distributor to solve the problem of determining the optimal volume of goods to purchase that will maximize additional profit.

Key words: wholesale trade, volume of purchases, commodity stock, profit, optimization model in trade

JEL: L81, F14, F17.

Introduction

The ability to competently and effectively organize a trading business is a subtle art in which one cannot rely only on practical experience, common

[1] Doctor of Technical Sciences, Professor, Institute of Management Problems of the Russian Academy of Sciences, Moscow, Russia, e-mail: al.zalozhnev@yahoo.com, ORCID: 0000-0001-7213-5519

[2] Doctor of Economics, Professor, Financial University under the Government of the Russian Federation, Moscow, Russia, e-mail: dchistov@fa.ru, ORCID:0009-0004-9202-2473

sense and intuition. Developers of the AGORA B2B platform claim that about 90% of errors and delays in business processes in the wholesale trade are associated with the so-called human factor. If some link in the chain to ensure the necessary inventory, accounting, labeling, location, pricing, and sales registration is implemented in the "manual mode", then failures are inevitable, which can seriously harm the trading business (How to increase…, 2020).

In this regard, introducing decision support tools into the management practice of trading companies based on the use of statistical and economic-mathematical methods, back calculations, artificial intelligence and machine learning (Odintsov, Urintsov, Mamedova, 2021; Urintsov, Dik, Larionov, 2018) is of great importance.

Theoretical and practical aspects of applying the methods of basic business mathematics in solving real problems of operating companies, according to various business scenarios, are considered in a significant number of works, for example, quite fully and in detail in (Clendenen, Salzman, 2014; Saunders, Emeritus, 2018; Carman, Saunders, 1986). Using these methods allows employees of trading companies to make informed decisions in the field of pricing, determining the optimal volumes of purchases and sales, and managing the assortment of goods. At the same time, it should be understood that with the various methods and approaches aimed at improving the efficiency of trading companies, in each specific situation, the task is to find adequate methods that most accurately reflect the specific aspects of a complex and diverse business (Kingsnorth, 2019).

Procurement optimization is understood as the process of regulating procurement activities that reduces costs by saving on the purchase and storage of inventory. The saved funds can be used to expand the production or service area in which the trading enterprise operates.

In (Zalozhnev, Chistov, 2022), the authors of the article proposed optimization models to support decision-making on the volume of purchases and the level of retail prices set by the wholesale company depending on the manufacturer's selling prices and the demand for various volumes of wholesale purchases. The models were built based on the assumption that both the manufacturer and the wholesaler influence pricing by pursuing a certain pricing policy. As a result, two tasks were set and solved to determine the optimal volume of wholesale purchases and sales, depending on supply and demand, as well as changes in the level of purchase and selling prices, for two cases: when the end buyer is either informed or not informed about producer prices.

The focus of this paper is on models and methods to support decision-making on the volume of purchases, depending on the assessment of upcoming retail sales.

In the daily practice of business entities, it is often necessary to determine the volume of purchases of certain goods. At the same time, it is necessary to consider the possibility of deviations from previous plans and to prevent negative consequences resulting from purchase of excess volumes of goods. The main problem is that the amount of excess must be optimal. This is necessary to avoid overstocking warehouses, freezing cash assets, or running out of stock to meet customer needs resulting from over- or under-purchases.

As a rule, in the decision process for the volume of purchases within an economic entity, the interests of two groups of employees collide: purchasing managers (sales) and employees of the financial department (financiers). The former seeks to increase the volume of purchases, based on the desire to have enough goods in stock and other considerations, which will be discussed below (in particular, due to possible changes in wholesale prices of the manufacturer). The latter seeks to reduce this volume, based on the fact that the liquidity of the company's assets will decrease if the purchased product is not sold in the period in which it is intended to be sold.

The main purpose of this paper is to propose mathematical models, the decision-making by the distributor on the volume of purchases is reached depending on the assessment of the upcoming retail sales, taking into account the dependence of the selling price of the manufacturer from the volume of purchases of the wholesaler (exclusive distributor, dealer), as well as the dependence of the retailer's selling price on the volume of purchases of the distributor from the manufacturer, while ensuring maximum additional profit. At the same time, within the framework of the study, the restrictive condition is observed that retailers cannot buy goods directly from the manufacturer, but only from a wholesaler.

General prerequisites

To build a decision-making model, consider an economic system consisting of three types of business entities:

- manufacturer,
- wholesaler (exclusive distributor or dealer),
- retailers.

We will proceed from the fact that retailers cannot purchase goods directly from the manufacturer, only from a wholesaler. The system trades goods of a single type. The minimum volume *V* of goods that can be purchased by a wholesale buyer from a manufacturer is equal to one.

Model I - manufacturer's selling price does not depend on the volume of purchases exclusive distributor (dealer).

Consider the following simulated situation. The distributor organization needs to plan the volume *V* of purchases of goods in the period *T + 1* if the volume of purchases of goods in the period *T - 1* is known, which constitutes V_0. During that time, the goods were fully sold. Period *T* is not considered, since it has not yet been completed and there is still no complete information about it: for example, the final volume of sales is unknown. Sales managers intend to increase the volume of purchases in the period *T + 1*, bringing it to the value *V*. The dealer purchases goods at the manufacturer's selling price *b(V)*, and sells them to customers at the retail price *q*.

For the purposes of this problem, the value of *V* is assumed to be continuous, which can be interpreted either as a small cost per unit of goods compared to the minimum volume of purchases (*V = 1*), or as the presence of a continuous scale of discounts to the manufacturer's selling price *b(V)* depending on the achieved volume of dealerships purchases *V*.

Suppose that the manufacturer's selling price depends on the volume of purchases of the exclusive distributor V and has the form shown in Figure 1.

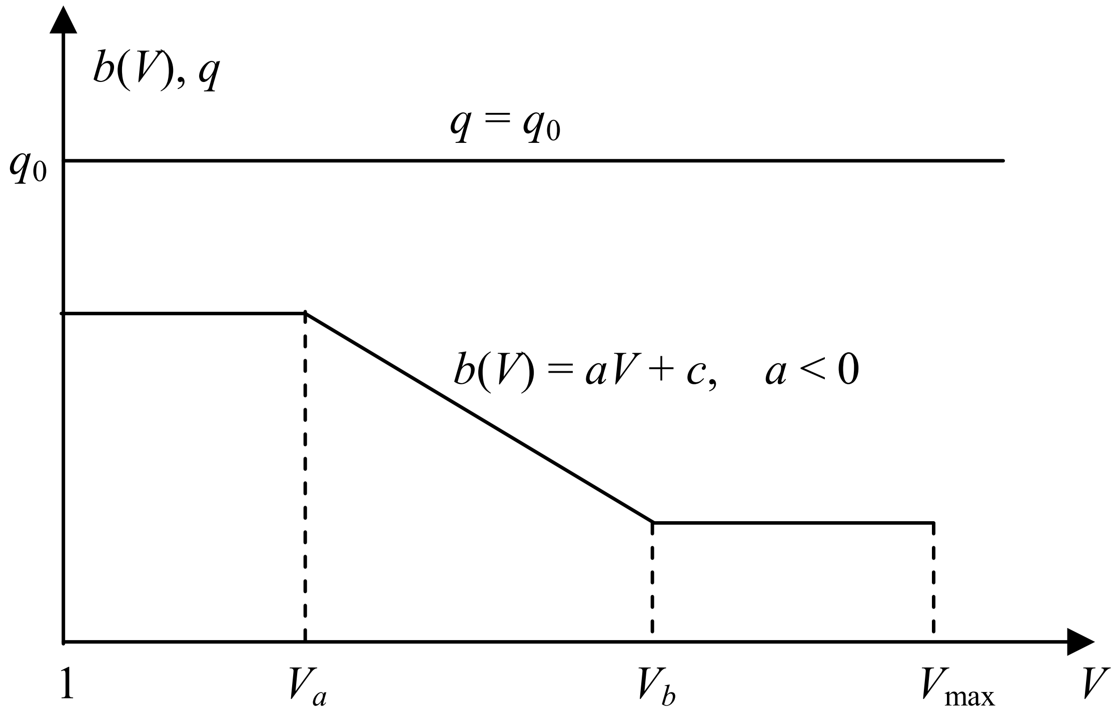


Figure 1. Influence of the volume of purchases on the manufacturer's selling price

Function $b(V)$, shown in Figure 1, can be meaningfully explained as follows. On the section [1, V_a] the manufacturer's selling price is constant, that is, the manufacturer's price does not respond to changes in the volume of purchases by dealers. On the section [V_a, V_b] the selling price decreases linearly:

$b(V) = aV + c$,

where $a < 0$.

Thus, the manufacturer seeks to increase dealer purchases by reducing the selling price with an increase in the volume of purchases. On the section [V_b, V_{max}] the manufacturer's selling price does not change, due to the fact that it is limited by the level of its variable costs. The value of V_{max} is limited by the production capabilities of the manufacturer or by the quota set for a particular dealer. In this case, we will proceed from the fact that the retail dealer price q is constant and does not depend on the value of V: $q = const = q_0$, $q > b(V)$ (see Fig. 1).

If everything purchased by the dealer in period T in volume V at price $b(V)$ is sold in the same period at price q, then the dealer's profit received in this period will be expressed by the ratio:

$P(V) = (q – b(V))\, V.$

If in the period T the volume of purchases is V, and the sales volume for this period is V_r ($V_r < V$), then the dealer's profit from sales will be

$P(V) = (q – b(V))\, V_r – b(V)\, (V – V_r) = qV_r – b(V)V$.

At first, consider the case when $V_0 \in (V_a, V_b]$. If, as we agreed, the function $b(V)$ is linear and has the form:

$b(V) = aV + c$, where $a < 0$,

then the function $P(V)$ can be transformed to the form:

$P(V) = qV_r – (aV + c)\, V = –aV_2 – cV + qV_r.$

Because $a < 0$, the graph of the function $P(V)$ is a parabola with branches pointing upwards. This function on $V \in (– \infty, + \infty)$ has a minimum. To find this minimum, we equate the derivative dP/dV to zero:

$$\frac{dP}{dV} = -2aV - c = 0,$$

where

$V_{ext} = -s/2a$.

Let us consider the possible locations of the points V_0, V_a, V_b and the points $V_{ext} = V_{ext1}$, V_{ext2}, V_{ext3}, V_{ext4} (see Fig. 2).

1) If the point V_{ext} is located on the V axis in the position V_{ext1} or V_{ext2}, then because for the desired optimal value V^* the ratio $V^* \geq V_0$ must be

satisfied (since we are talking about an increase in the volume of purchases), then we choose V^* as V_0: $V^* = V_0$.

2) If the V_{ext} point is located on the V axis at the V_{ext3} position, then $V^* = arg\ max\ \{P(V_0), P(V_b)\}$.

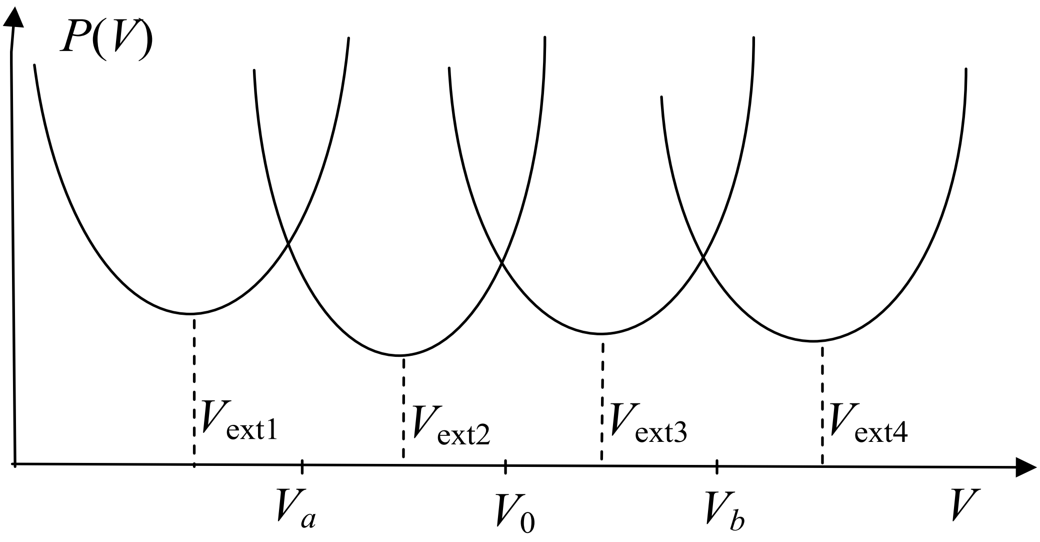


Figure 2. Possible location of points minimum profit

2) If the V_{ext} point is located on the V axis at the V_{ext3} position, then $V^* = arg\ max\ \{P(V_0), P(V_b)\}$.

3) If the point V_{ext} is located on the axis V as V_{ext4}, then, obviously (see Fig. 1), $V^* = V_0$.

From the options (1) - (3), it can be seen that only in case (2), when V_{ext} is located as V_{ext3} ($V_{ext} = V_{ext3}$), the value V^* can take on a value different from V_0. This case, of course, requires consideration, but since in all other cases ($V_{ext} = V_{ext1}, V_{ext2}, V_{ext4}$) the value V^* takes the value V_0, then as an operational practical solution in the situation described in the above model, $V^* = V_0$.

Consider an even simpler case, when $b(V)=b_0$, i.e. the producer's selling prices are constant or, the same, $V \in [1, Va]$ or $V \in [V_b, V_{max}]$. In this case, $P(V) = qV_r - b_0V$.

Since also $q = const$, the maximum of the function $P(V)$ is reached either at the maximum V_r or at the minimum V.

Let us consider the decision-making process by the dealer about the choice of the value V of the volume of purchases for the next ($T + 1$) period in more detail.

Suppose that in the last completed period of economic activity – period (*T - 1*) – the dealer sold the goods in the volume V_0. Let us assume that the managers of the purchase (sale) plan to purchase goods in volume *V* for sale in the period (*T + 1*). Then, depending on the ratio between the volume of consumer demand V_r in the period (*T + 1*) and the volume of purchases *V* from the manufacturer in the period (*T + 1*), the amount of profit *P (T +1)* can take the following values:

a) $V_r >= V$, i.e., all goods purchased in volume *V* in the period (*T + 1*) will be sold. Then

$P\ (T + 1) = qV – b_0V = (q–b_0)\ V.$

b) $V_r < V$ – the goods will be sold only in the volume V_r. Then

$P\ (T + 1) = (q – b_0)\ V – q\ (V – V_r) = qV_r – b_0V.$

Where the value of profit *P* in the period (*T + 1*) depending on the volume of demand in this period – V_r as a parameter, and the volume of purchases *V* as an argument – can be represented as:

$$P(V)=\begin{cases}(q-b_0)\ V,\ V_r \geq V\\ (q-b_0)\ V-q(V-V_r),\ V_r<V\end{cases}.$$

In order to decide to increase the volume of purchases in the period *T + 1* compared with the period *T - 1*, the financial department can be guided by information on the amount of additional profit that can be gained from an increase in the volume of purchases and, accordingly, sales in the period *T + 1* compared with period *T - 1*. If in the period *T - 1* the volume of sales is equal to the volume of purchases and amounts to V_0, and the profit, respectively, amounts to

$P\ (T – 1) = (q – b_0)\ V_0$,

then the additional profit *ΔP* from an increase in the volume of purchases from V_0 to *V* under the condition a) $V \leq V_r$ will be

$ΔP = (q – b_0)\ V – (q – b_0)\ V_0 = (q – b_0)\ (V – V_0),$

and under condition b) $V > V_r$ will be

$ΔP = (q–b_0)\ V – q(V–V_r) – (q–b_0)\ V_0 = (q–b_0)\ (V–V_0) – q(V–V_r)$,

or, which is the same, we write in the final form:

$$\Delta P=\begin{cases}\Delta P_1=(q-b_0)(V-V_0),\ V \leq V_r\quad (a)\\ \Delta P_1-q(V-V_r),\ V>V_r\quad (б)\end{cases}. \tag{1}$$

Let us consider the case when employees of the financial department, based on accounting requirements about the need for a pessimistic assessment, choose the sales volume V_0 achieved in the period *T - 1* as the guaranteed sales volume in the period *T + 1*. From the point of view of our model, this means that in relationship (1), V_r should be replaced by V_0, excluding case (a) from consideration, since in this case $V > V_0$ according to the meaning of the problem, only case (b) should be considered. Then we have:

$$\Delta P = (q - b_0)(V - V_0) - q(V - V_0) = -b_0(V - V_0).$$

It can be seen from the middle part of this equation that since $|q| > |q - b_0|$ in this case the negative component of the increase in profit grows faster than its positive component.

Based on the result, it can be concluded that in the case when:

- an increase in dealer purchases from the manufacturer does not lead to a decrease in the manufacturer's selling prices, i.e. when $b(V) = b_0 = const$,
- dealer selling prices q are constant ($q(V) = const$),
- in the period $T - 1$, the dealer's sales amounted to V_0,

then in this case, any increase in the volume of dealer purchases compared to V_0 in the period $T + 1$ is inappropriate.

The validity of the last statement can be limited in the following substantive cases:

a) when in the current period T, there is a significant increase in sales compared to the period $T - 1$;

b) when an increase in the volume of dealer purchases from V_0 to V leads to a significant decrease in the share of fixed overhead costs per unit of goods purchased by the dealer in kind and / or in value terms. This case is essentially equivalent to the case considered above, when the manufacturer's selling price $b(V)$ decreases with an increase in the volume of dealer purchases V. However, as shown above, for this case, with a single exception, the same meaningful conclusions are valid as for the case when $b(V) = b_0$.

Let us now consider relationship (1) from the manager's point of view. For case (b) – ($V > V_r$) and rewrite it in the form:

$$\Delta P = (q - b_0)(V - V_0) - q(V - V_r).$$

From the manager's point of view (optimistic point of view), in contrast to the pessimistic point of view of the finance department, the relationship $V_r > V_0$ is satisfied. Moreover, the manager usually feels confident to etermine a fairly accurate estimate of the value of V_r. Since the amount of additional profit ΔP reaches a maximum at $V = V_r$, and begins to decrease at values of V such that $V > V_r$, managers will obviously recommend that in the period $T + 1$ the volume of purchases be increased from V_0 to V_r.

Let us consider in more detail the process of searching for the volume of purchases, in which purchasing and sales managers and employees of the financial department participate.

According to the manager, by increasing the volume of purchases and, accordingly, sales from V_0 to V_1 (V_1 =< V_r), the dealer firm will receive additional profit in the amount of:

$\Delta P = (q - b_0)(V_1 - V_0)$.

According to the financier, the dealer firm in the period $T + 1$ will be able to sell goods only in the volume V_2: $V_0 < V_2 < V_1$. And, consequently, having purchased goods in volume V_1, the dealer firm will receive additional income (additional loss) in the amount of

$$\Delta P = (q - b_0)(V_2 - V_0) - q(V_1 - V_2) \qquad (2)$$

Since the financial risks lie with the financier (financial department), that person, as a decision maker, will follow, for example, such a strategy: agree with the manager and allow an increase in the volume of purchases above the value V_2, which seems guaranteed, to the volume V^* so that the amount of additional profit, which, according to the financier, has a maximum at $V = V_2$, would become equal to zero. In this case, if the financier turns out to be right, the dealer firm will not receive additional losses in the period $T + 1$, and if the manager turns out to be right, the firm will receive additional profit. Replacing V_1 with V^* in relationship (2) and equating it to zero, we obtain the relationship for determining the value of V^*:

$(q - b_0)(V_2 - V_0) - q(V^* - V_2) = 0$,

where

$$V^* = V_2 + \left(1 - \frac{b_0}{q}\right)(V_2 - V_0),$$

thus, we obtain the desired expression for determining the optimal value of the volume of purchases V^*, which provides the maximum additional profit.

Model II - manufacturer's selling price depends on the volume of purchases of the exclusive distributor (dealer).

Consider another meaningful model (model II), based on the fact that the dealer’s selling price q is not a constant value, but it is determined by the manufacturer and depends on the volume of the dealer's purchases from the manufacturer.

When constructing the model, we will proceed from the following prerequisites:

1) The dependence of the manufacturer's wholesale prices on the volume of dealer purchases is known - *b(V)*.

2) The dealer's selling prices *q* are determined by the manufacturer and are related to the manufacturer's selling prices *b* for a given volume of purchases *V* by the relationship:

$q(V) = (1 + \alpha)\, b(V),\ \ \alpha > 0.$

3) The manager gives an accurate estimate of the value of V_r the maximum sales volume in the period *T + 1*, and $V_r > V_0$, where V_0 the sales volume in the period *T - 1*. It is necessary to determine the volume of purchases $V = V$ ***, at which the maximum additional profit is achieved *ΔP*, which is given by a relationship similar to relationship (2), taking into account the assumptions just made:

$\Delta P = [q(V) – b(V)]\,(V – V_0) – q(V)\,(V_r – V).$

4) Suppose that the dependence of the wholesale price of the manufacturer on the volume of dealer purchases is linear and has the form:

$b(V) = aV + c$, where $a < 0$.

Whence we obtain the following relationship for the value of additional profit *ΔP*:

$\Delta P = [(1+\alpha)b(V) – b(V)](V–V_0) – (1+\alpha)b(V)(V_r–V) =$

$= b(V)\, [(1+2\alpha)V– (1+\alpha)V_r –\alpha V_0].$

Since from the meaning of the problem $V > V_r$, V_0, then $(1+2\alpha)V – (1+\alpha)V_r –\alpha V_0 > 0$ and the graph of the function $\Delta P(V)$ is a parabola with its branches pointing down and reaching its maximum at interval $[–\infty,+\infty]$.

Since $b(V) = aV + c$, where $a < 0$, then, by introducing the coefficient a_1 so that $a_1 = –a$ ($a_1 > 0$), we obtain

$b(V) = –a_1V + c$,

where

$\Delta P = (–a_1V + c)[(1 + 2\alpha)V – (1 + \alpha)V_r – \alpha V_0] = –a_1(1 + 2\alpha)V_2 + a_1[(1+\alpha)V_r + \alpha V_0]V + (1+2\alpha)\, cV+ ...$ are terms independent of *V*.

To find the value of *V* = *V*, we equate the derivative $d\Delta P/dV$ to zero and get

$d\Delta P/dV = –2a1(1 + 2\alpha)V + a1[(1 + \alpha)Vr + \alpha V_0] + (1 + 2\alpha)\, c = 0,$

where

$$V^* = \frac{a_1[(1+\alpha)V_r + \alpha V_0] + c(1+2\alpha)}{2a_1(1+2\alpha)} \qquad (3)$$

The result obtained means that for the given price dependences *b(V), q(V)*, the known volume of sales of goods by the dealer in the period $T\text{-}1 – V_0$ and the estimate of the volume of sales given by managers in the period $T + 1\text{-} V_r$, we obtained the value of the volume of dealer purchases V^* $(V^*>V_r)$, at which the maximum value of additional profit ΔP is reached in the period $T + 1$.

Conclusion

The paper considers models and algorithms for making decisions regarding the volume of purchases of goods by a wholesale organization (dealer, distributor) from the manufacturer based on information on the volumes of purchases of this product in the previous period, provided that the price set by the manufacturer depends on the volumes of purchases, and the volumes of purchases limited by the manufacturer's capabilities or by a quota set by the manufacturer for a particular wholesaler.

The models proposed in the article are built on the assumption that the company trades only one product. Since in the real conditions of the activity of a trade organization, one has to deal with a variety of goods, it is assumed that the proposed models and decision-making algorithms must be applied separately to each type of goods or homogeneous groups of goods.

The solution of such a problem implies the need for a large number of calculations, in which all the necessary initial numerical data are fully presented in the information bases of the accounting and analytical systems of trade organizations. This provides an opportunity for software implementation of the proposed algorithms in the environment of the relevant information systems. Possible options for embedding economic and mathematical models and methods in the environment of existing information systems are considered by the authors of this article in (Zalozhnev, Chistov, Shuremov, 2022; Zalozhnev, Loktionov, Makeev, Chistov, Shuremov, 2022).

References

Carman, R. A., Saunders, H. M. (1986). *Mathematics for the Trades: A Guided Approach*. Hoboken, New Jersey, U.S.: J. Wiley.

Clendenen, G., Salzman, S. (2014). *Business Mathematics*. London, UK: Pearson.

How to increase the efficiency of wholesale trade with minimal costs, or B2B 2.0 (2020). General Director. February 2020 URL: https://www.gd.ru/articles/10739-kak-povysit-effektivnost-optovoy-torgovli-s-minimalnymi-zatratami-ili-b2b-20-adv (Date of access: 09/30/2022).

Kingsnorth, S. (2019). *Digital Marketing Strategy: An Integrated Approach to Online Marketing*. 2nd Edition. London, UK: Kogan Page.

Odintsov, Boris E., Urintsov, Arkadiy I. and Mamedova, Natalia A. (2021). *"Business process performance management of enterprises in terms of inverse calculations"*. AIP Conference Proceedings 2410, 020015.

Saunders, H., Emeritus, R.C. (2018). *Mathematics for the Trades: A Guided Approach (What's New in Trade Math)*. 11th edition. London, UK: Pearson.

Urintsov, Arkadiy I., Dik, Vladimir V., Larionov, Andrey S. (2018). Development of decision support systems through the contradictions of informational society. *International Journal of Information and Decision Sciences*, *Vol. 10, no. 4*, Published Online: October 9, pp 279-296.

Zalozhnev, A. Yu., Loktionov A. E., Makeev V. V., Chistov D. V., Shuremov E. L. (2022). *Upravlenie I kommunikatcionnye tekhnologii vzaimootnoshenii s kontragentami: Monograph - M*: Ed. House of MISiS, 143 p.

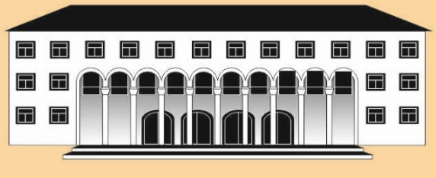
Zalozhnev, Alexey, Chistov, Dmitry. (2022). Models for making decisions on the volume and price of purchases in wholesale trade. *Business Management*, №2, pp. 59–70.

Zalozhnev, A. Yu., Chistov, D. V., Shuremov, YE. L. (2022). *Informatsionnyye tekhnologii upravleniya vzaimootnosheniyami s kliyentami*. Moskva: PMSOFT, ISBN 978-5-903 l83-16-6, 124 p.

ISSN 0861 - 6604
ISSN 2534 - 8396

БИЗНЕС управление 1/2023

BUSINESS management



BUSINESS management 1/2023

PUBLISHED BY
D. A. TSENOV ACADEMY
OF ECONOMICS - SVISHTOV

1/2023

The printing of the issue 1-2023 is funded with a grand from the Scientific Research Fund, Contract KP-06-NP4/75 /16.12.2022 by the competition “Bulgarian Scientific Periodicals - 2023“.

Submitted for publishinq on 29.03.2023, published on 30.03.2023, format 70x100/16, total print 80

BUSINESS management

D. A. Tsenov Academy of Economics, Svishtov

Year XXXIII * Book 1, 2023

CONTENTS